10.5 Permutations and Combinations

Warmup

How many different ways can the letters be arranged?

1. FLOWER 720
2. STUDY 120
3. POP 3
4. SEE 3
5. PEGGY 60
6. LEVEL 30
7. MISSISSIPPI 34,650
8. APPROPRIATE 1,663,200

10.5 Permutations and Combinations

Multiplication, Addition & Complement Principles

How many 3 digit numbers have at least one 7?

7	(9)	(9)		7	7	(9)		7	7	7	
(8)	7	(9)	225 ways	(8)	7	7	26 ways				1 way
(8)	(9)	7		7	(9)	7					

= 252 ways

A better way: All 3 digit #'s minus all 3 digit #'s without a 7

900 - (8)(9)(9) = 252 ways

10.5 Permutations and Combinations

Multiplication, Addition & Complement Principles

How many 3 digit numbers have at least one 7?

7	(9)	(9)		7	7	(9)		7	7	7	
(8)	7	(9)	225 ways	(8)	7	7	26 ways				1 way
(8)	(9)	7		7	(9)	7					

= 252 ways

Practice -

1. How many 4 digit #'s contain at least one 8 or 9?
Hint:
(all possible)-(#'s w/o 8 or 9)

5,416

2. How many 4-digit #'s contain 2 or more zeros?

252

10.5 Permutations and Combinations

Combinations

- **Definition** - An arrangement of objects in which order is NOT important.

Example 1: The combinations of 2 letters in the word JULY.

6 ways - JU, JL, JY, UL, UY, LY

Example 2: The combinations of all the letters in the word JULY.

only 1 way - JULY

10.5 Permutations and Combinations

Combinations Formula

The number of combinations of n objects taken r at a time, where $r \leq n$, is given by:

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

Order does not matter!!!!

Example 1: The combinations of 2 letters in the word JULY.

$$ {}_4C_2 = \frac{4!}{2!(4-2)!} = 6 \ ways $$

Example 2: The combinations of all the letters in the word JULY.

$$ {}_4C_4 = \frac{4!}{4!(4-4)!} = 1 \ way $$

10.5 Permutations and Combinations

Combinations Formula

The number of combinations of n objects taken r at a time, where $r \leq n$, is given by:

$$_nC_r = \frac{n!}{r!(n-r)!}$$

Order does not matter!!!!

Practice

1. You order a sandwich at a restaurant. You can choose 2 side dishes from a list of 8. How many combinations of side dishes are possible?

$$_8C_2 = \frac{8!}{2!(8-2)!} = 28$$

10.5 Permutations and Combinations

Permutations

There are 8 chair and 5 students.
How many different seating arrangements are possible?

$$Total = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = \frac{8!}{3!} = \frac{8!}{(8-5)!}$$

$$= 6{,}720 \ ways$$

$$_nP_r = \frac{n!}{(n-r)!}$$

Order of objects matter

Combinations

There are 8 players and 5 are chosen to start on the team.
How many combinations can be formed?

$$Total = \frac{8!}{(8-5)!(5)!} = 56 \ ways$$

Order doesn't matter

$$_nC_r = \frac{n!}{r!(n-r)!}$$

10.5 Permutations and Combinations

Combinations

There are 8 players and 5 are chosen to start on the team.
How many combinations can be formed?

$$Total = \frac{8!}{(8-5)!(5)!} = 56 \ ways$$

Order doesn't matter

$$_nC_r = \frac{n!}{r!(n-r)!}$$

Practice -

1. From a list of 12 books, how many groups of 5 can be selected?

792

2. How many baseball teams of 9 can be formed from 14 players?

2,002

10.5 Permutations and Combinations

Combinations

From a deck of 52 cards, how many ways can 4 cards be drawn so that all 4 are face cards? No face cards? At least one face card?

4 face cards:

$${}_{12}C_4 = 495$$

No face cards:

$${}_{40}C_4 = 91{,}390$$

At least 1 face card:

$${}_{52}C_4 - ??? =$$

$${}_{52}C_4 - {}_{40}C_4 = 179{,}335$$

Practice -

1. From a deck of 52 cards, how many different 3 cards can be drawn so they are all diamonds? No diamonds? At least one diamond?

3 diamonds: 286

No diamonds: 9,139

At least 1 diamond: 12,961

10.5 Permutations and Combinations

Combinations

From a deck of 52 cards, how many ways can 4 cards be drawn so that all 4 are face cards? No face cards? At least one face card?

4 face cards:

$${}_{12}C_4 = 495$$

No face cards:

$${}_{40}C_4 = 91{,}390$$

At least 1 face card:

$${}_{52}C_4 - ??? =$$

$${}_{52}C_4 - {}_{40}C_4 = 179{,}335$$

Practice -

1. There are 4 algebra books, 2 geometry books, and 3 pre-calc books. All the books are from different authors. In how many ways can you arrange the books in a shelf if books in the same subject must stay together?

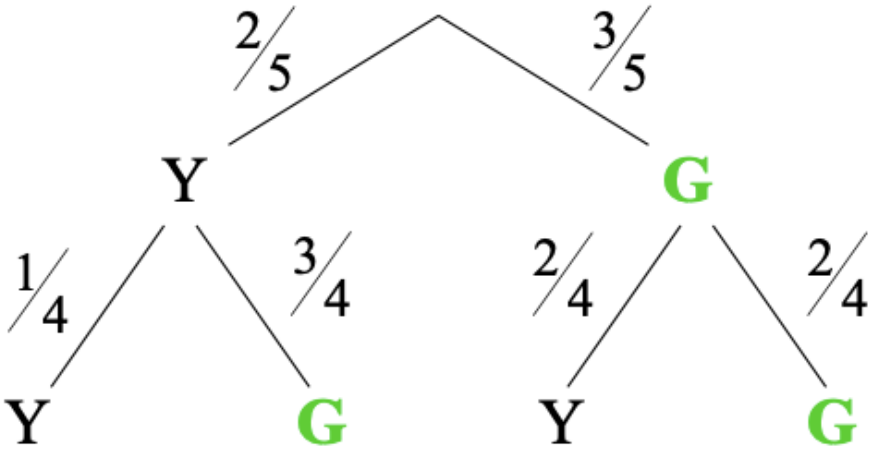
(3!)(4!)(2!)(3!) = 1,728

Conditional Probability

10.2 Independent and Dependent Events

Probability

There are two yellow balls and three green balls in a jar.
Two are chosen randomly without replacement.



1) $P(2nd\ a\ yellow \mid 1st\ was\ green)$

$$= \frac{2}{4} = \frac{1}{2}$$

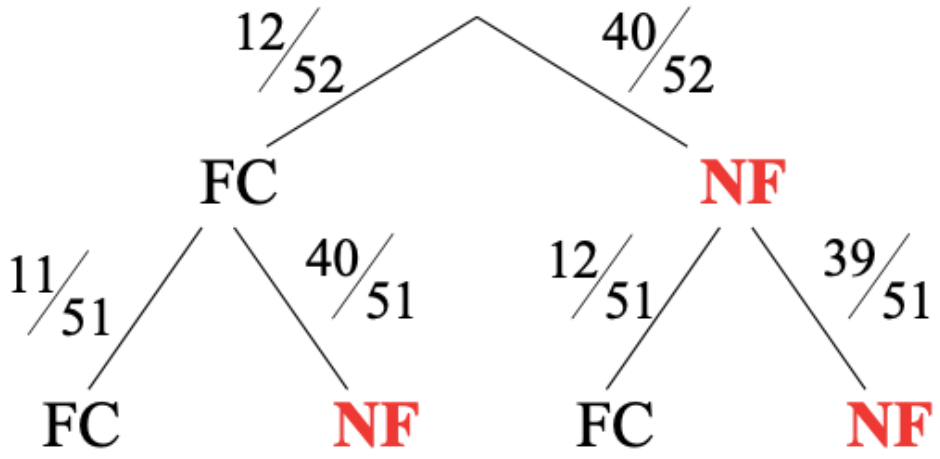
2) $P(3rd\ yellow \mid 1st\ two\ green)$

$$= \frac{2}{3}$$

10.2 Independent and Dependent Events

Probability

Two cards are chosen from a deck.
Draw a tree diagram showing the probability of face card vs. non-face card.



1) $P(FC \mid 2\ NF\ chosen)$

$$= \frac{12}{50} = \frac{6}{25}$$

2) $P(NF \mid 1FC\ and\ then\ 1NF)$

$$= \frac{39}{50}$$

10.2 Independent and Dependent Events

Conditional Probability

1) One card is chosen from one standard deck that has been shuffled (52 cards total). Determine the following:

a. P(face card | black card) $\frac{6}{26} = \frac{3}{13}$

b. $P((diamond \cup king) \mid red\ cards)$

$$\frac{13 + 2 - 1}{26} = \frac{14}{26} = \frac{7}{13}$$